Week 2: Logit, Probit, and Beyond

POLI803

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Week 2

University of South Carolina

Outline

- Binary outcome and logit models
- Estimation and interpretation
- Calculating predicted marginal effects and showing effect plots
- Rare event logit extension

Limited dependent variable models

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  - vote choice between Con., Lib. Dem., or Labour [nominal]
  - number of terrorist attacks [count]
  - $Y >$ threshold (0) [censored]
  - percentage [0–100]
- These variables are called **limited dependent variables** (categorical/restricted range).

  We cannot & should not use a linear model to analyze limited DV!

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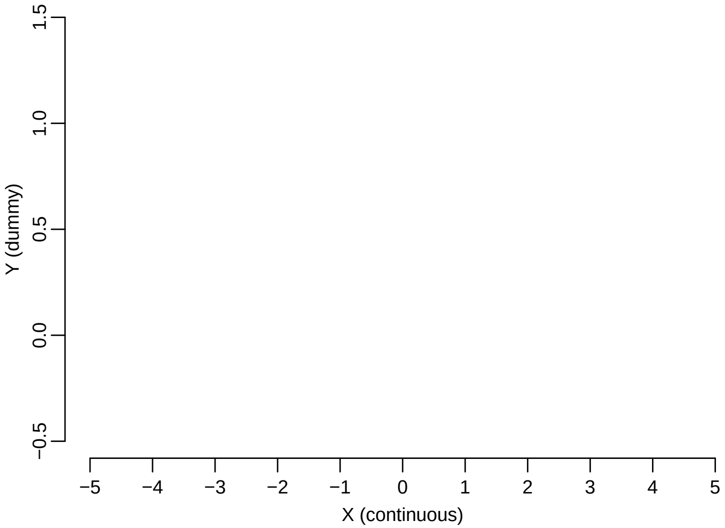
- straight lines from LM would be a poor representation of the X-Y relationship when Y is a limited DV;
  - implausible predicted values
  - marginal effect forced to be constant
- (quadratic / log curves cannot handle this, either)
- We need **Generalized linear models (GLM)**: a flexible generalization that allows for outcome variables to have arbitrary distributions or an arbitrary function of the response variable (the **link function**) to vary linearly with the predictors

We will first consider the case of a binary/dummy DV.

Scatterplot for binary/dummy DV



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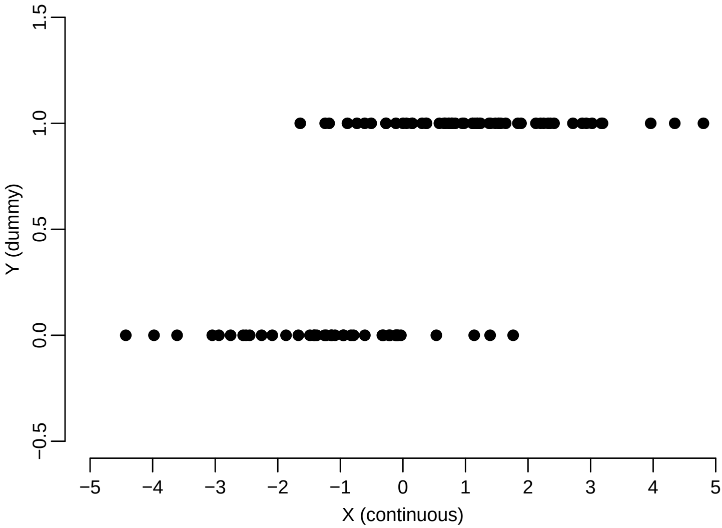

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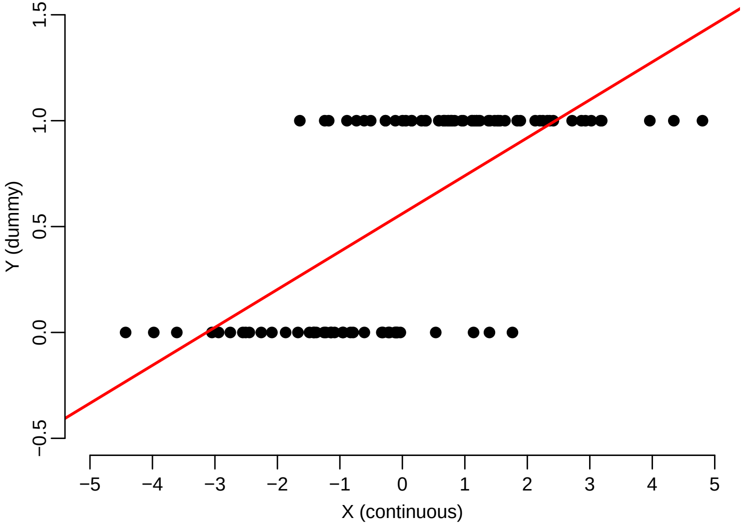
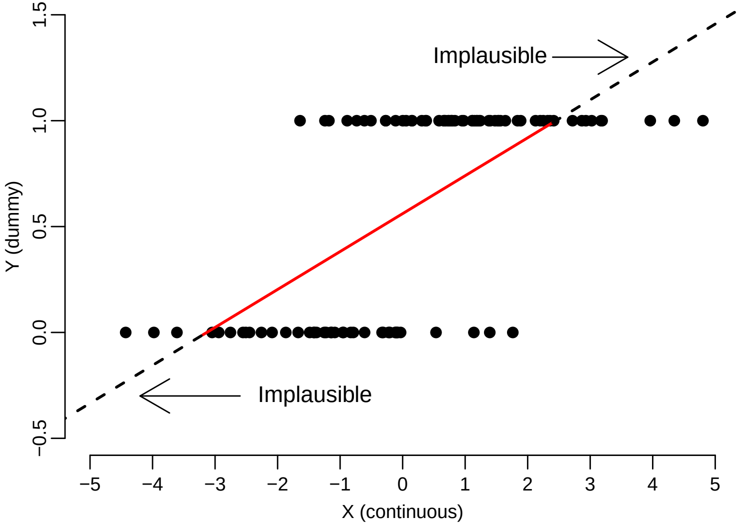


(1) Implausible predictions



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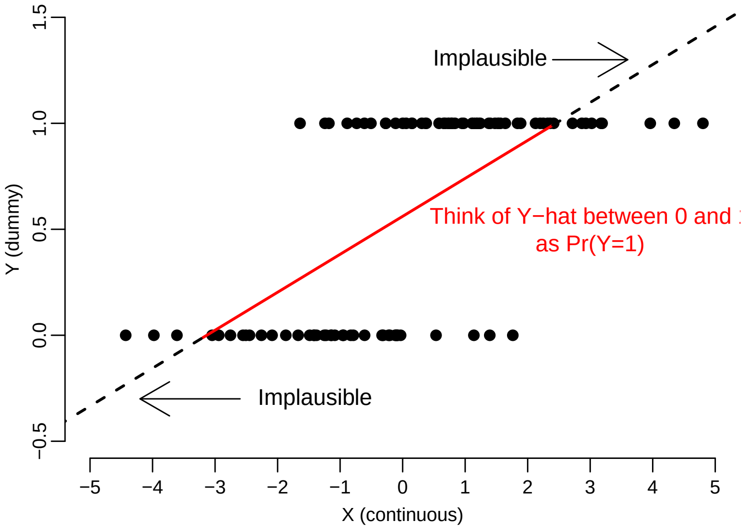
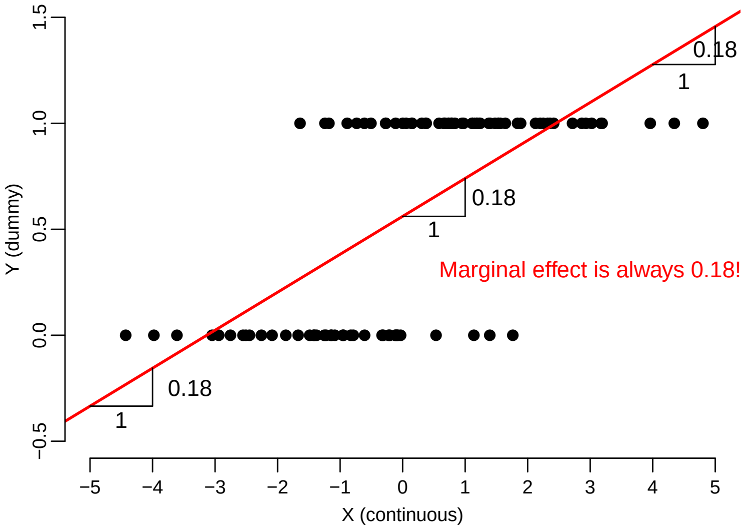
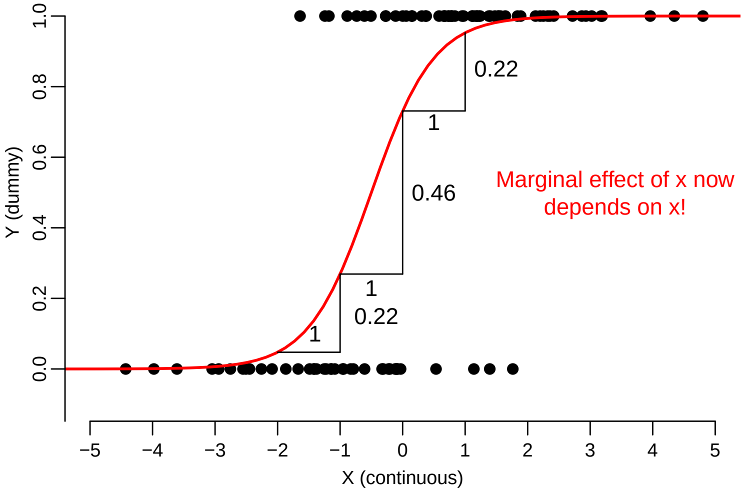


(2) Constant marginal effect of X



Fit an S-shaped curve



Fit an S-shaped curve

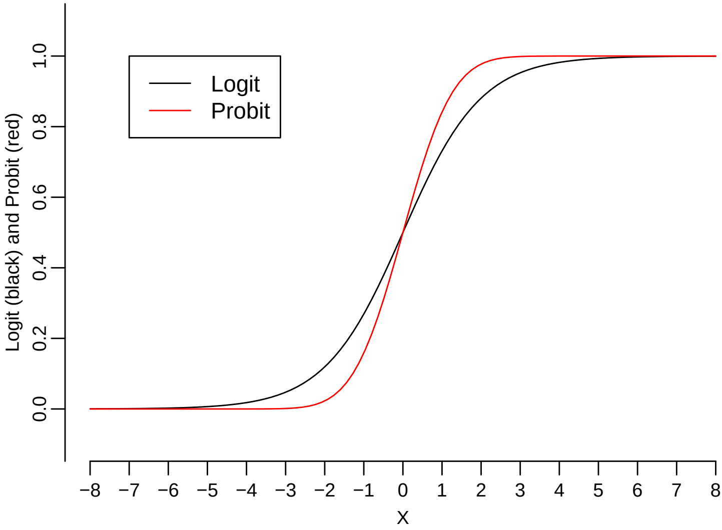
When we fit an S-shaped curve (instead of a line):

- No implausible predicted values;
- Predicted values can be thought of as **latent probabilities** $Pr(Y = 1)$ or $\hat{P}$;
- Marginal effect of X ($\frac{\partial \hat{P}}{\partial X}$) depends on the values of X;
- (Marginal effect of X also depends on the values of the other covariates included in the model!).

Fit an S-shaped curve

- If we run **logit regression** (logistic regression, logit model), we fit a logistic curve.
- If we run **probit regression** (probit model), we fit a probit curve.
- Logit and probit curves are very similar in shape, so you can just run one, not both.

Logist and Probit Curves



Logit regression

Logit regression can be represented as:

$$Y^* = \alpha + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \cdots + \beta_k X_k$$
$$\hat{P} = \Lambda(Y^*)$$

where $\Lambda(x) = \frac{1}{1+\exp(-\beta x)}$ is called the **link function**

- Y^* = latent utility (propensity).
- Y^* can range between $-\infty$ and ∞, but $\hat{P}$ ranges between 0 and 1.
- β_m shows **the marginal effect of X_m on Y^***, but NOT the effect of X_m on $\hat{P}$ itself.
- Yet, we are interested in **the effect of X_m on $\hat{P}$**.

Probit regression

Probit regression can be represented as:

$$Y^* = \alpha + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \cdots + \beta_k X_k$$
$$\hat{P} = \Phi(Y^*)$$

where $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left(-\frac{x^2}{2}\right) dx$

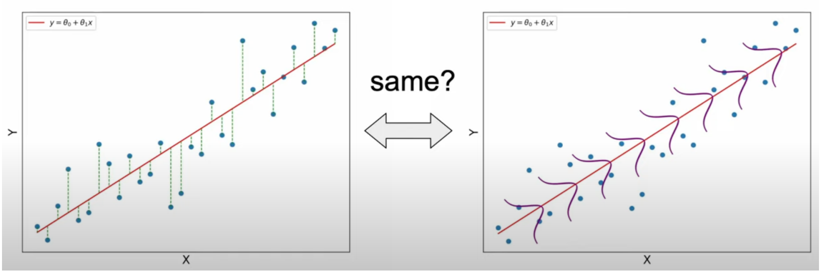
- Logit regression uses the logit link function, $\Lambda()$.
- Probit regression uses the probit link function, $\Phi()$.

The LPM model or Logit model?

- The linear probability model (LPM) fits a linear regression model to a binary response variable, often using OLS.
- OLS supporters:
  - LPM is the wrong but super useful model because changes (marginal effects) can be interpreted in the probability scale
  - OLS not always give nonsensical predictions
  - Causal inference: most causal inference techniques rely on OLS (2SLS and DiD)
- MLE supporters:
  - LPM is the wrong, period
  - If model fit and prediction accuracy are the goals, logit (and other MLE estimators) always win

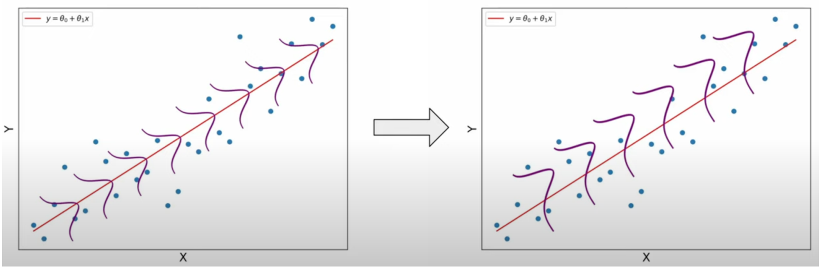
Difference between OLS and MLE

- Our old friend: Ordinary Least Squares (OLS)



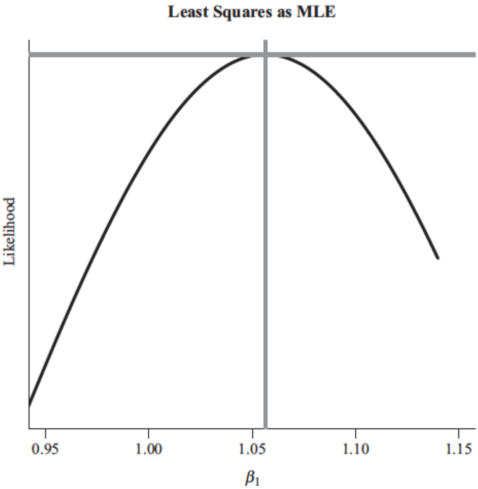
- OLS is a very special case of Maximum Likelihood Estimation that happens when "errors are normally distributed"

What to do if errors are *not* normally distributed?



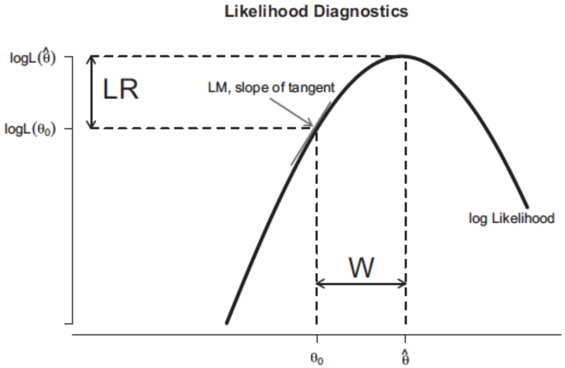
- Least squares method can't work anymore

Our new friend: Maximum Likelihood Estimation (MLE)



Our new friend: Maximum Likelihood Estimation (MLE)

- MLE: maximize the likelihood of observing θ given a probability distribution (e.g., Logit distribution)



MLE Estimator

- The logistic functional form:

$$\theta_i = \texttt{logit}^{-1}(x_i^T \beta) = \frac{1}{1+e^{-x_i^T \beta}} \qquad (1)$$

- Joint probability (the product of all conditional probability) for a Bernoulli random variable

$$\Pr(y \mid \theta) = \prod_{i=1}^{n} \theta_i^{y_i}(1-\theta_i)^{1-y_i} \qquad (2)$$

- Take the log-likelihood

$$\begin{aligned}
\log L(\theta \mid y) &= \sum_{i=1}^{n} [y_i \log \theta_i + (1-y_i)\log(1-\theta_i)] \\
&= \sum_{i=1}^{n} \left[y_i \log\left(\frac{1}{1+e^{-x_i^\top \beta}}\right) + (1-y_i)\log\left(1 - \frac{1}{1+e^{-x_i^\top \beta}}\right)\right] \\
&= \sum_{i=1}^{n} \left[-y_i \log(1+e^{-x_i^\top \beta}) + (1-y_i)\log\left(\frac{e^{-x_i^\top \beta}}{1+e^{-x_i^\top \beta}}\right)\right]
\end{aligned}$$

MLE Estimator

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- How to estimate this θ?

MLE Estimator

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- Newton Raphson approximation → R will do it for you, fortunately
  - If you're interested the hand derivation in math → read

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- Note: MLE estimation is not restricted by gaussian/normal anymore, given that the functional form of logit distribution has been identified by statisticians.

Do you remember? Bayesian estimation of θ

$$\underbrace{\xi(\theta|x)}_{\texttt{posterior dist.}} \propto \underbrace{f(x|\theta)}_{\texttt{data/Likelihood}} \underbrace{\xi(\theta)}_{\texttt{prior dist.}}$$

- Bayes just goes a little further by multiplying the likelihood function with a prior guess

Logistic Regression in Rare Events Data

- If we only have 20 events in our total 100 observations, can we still use logit?

Logistic Regression in Rare Events Data

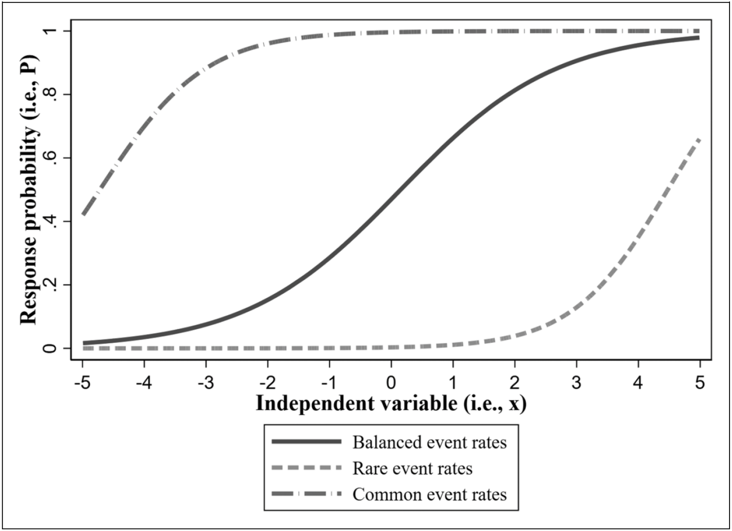

Figure 1. Effects of rare and common binary dependent variables on response probabilities.

The Problem

- Failed to capture reality
- Response probabilities increase drastically at the low values of x and remain near or at 1 through the high values of x. $\rightarrow$ steep increase of response probabilities at one end or the other of the spectrum of x values with mostly flat probabilities leads to inflation biases when using logit and probit models

Proposed Solutions: Firth logit (1993)

- Penalized MLE: a **penalty based on the Jeffreys prior** to the log-likelihood function. Instead of maximizing the usual likelihood, it maximizes the **penalized likelihood**:

$$\ell^*(\beta) = \ell(\beta) + \frac{1}{2} \log |I(\beta)|$$

- $\ell(\beta)$: the usual log-likelihood from logistic regression
- $I(\beta)$: the Fisher information matrix
- $\log |I(\beta)|$: a correction term that reduces bias

- Helps pull extreme estimates (especially those caused by rare events or separation) back toward more reasonable values.

Proposed Solution: ReLogit (King and Zeng, 2001)

- Use weighted MLE: a **prior correction** that reweights the predicted probabilities to reflect the true event prevalence in the population

Let:

- The reality: τ be the true fraction of 1s (events) in the population (e.g., 0.01),
- The model: $\bar{y}$ be the fraction of 1s in the sample (e.g., 0.20, if oversampled),
- $\hat{p}$ be the predicted probability from the rare events logistic regression.

Then the **corrected predicted probability** is:

$$\hat{p}_{\text{corrected}} = \left[\frac{\tau(1-\bar{y})}{\bar{y}(1-\tau)} \cdot \frac{\hat{p}}{1-\hat{p}}\right] \Bigg/ \left(1 + \frac{\tau(1-\bar{y})}{\bar{y}(1-\tau)} \cdot \frac{\hat{p}}{1-\hat{p}}\right)$$

Proposed Solution: ReLogit (King and Zeng, 2001)

- This correction adjusts the predicted odds to produce probabilities consistent with the actual (rare) base rate in the population.
- For example, if you build a model where 20% of the sample are events (e.g., coups), but in reality coups occur only in 1% of country-years $\rightarrow$ set $\tau = 0.01$ (true prevalence of the rare event)

The Strategy

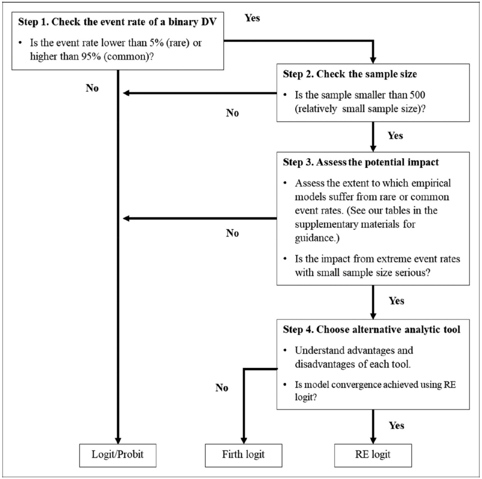

Figure 4. Decision tree.

Source: Woo, H. S., Berns, J. P., & Solanelles, P. (2023). How rare is rare? *Organizational Research Methods, 26*(4), 655–677.

Estimation in R

Running a logit / probit model is quite easy in R.

```
fit <- glm (y ~ x1 + x2 + x3...,
     data = dataset.name,
     family = binomial(link = logit))

fit <- glm (y ~ x1 + x2 + x3...,
     data = dataset.name,
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```

What's not quite easy is to interpret the results.

Interpretation

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- With logit model, β_1 merely shows the marginal effect of X_1 on Y^*, which is not the quantity of interest.
- With logit model, what we care is: $\frac{\partial \hat{P}}{\partial X_1}$, or the effect of X_1 on the probability $Y = 1$. (We care about the $\hat{P}$)
- Moreover, the marginal effect of X_1 on $\hat{P}$ differs depending on the value of X_1 itself as well as other Xs included in the model.

What to do after estimation

Three Steps

1. Produce a regression table using `stargazer`.
    - Identify the "best" model(s)
2. Discuss statistical significance and the **sign** (but not the size) of coefficients.
3. Graphically illustrate the **size** of the marginal effects (and discuss them in the text).
    - Do this for "interesting" and/or "representative" cases in your covariates

How to illustrate the marginal effect of X

$$\hat{P} = \Lambda(\alpha + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \cdots + \beta_k X_k)$$

1. Choose one X to focus on. Let's say we are interested in X_1.
2. Set the values of all the other Xs at their "interesting" and/or "representative" values (mean, median, minimum, maximum, etc.).
3. Effect plot: Graphically and numerically show the relationship between X_1 and $\hat{P}$ using the `effect` function.

Example: Titanic passenger survival

```
> head(td)
  survived                                            name pclass     age child
1        1                    Allen, Miss. Elisabeth Walton      1 29.0000 Adult
2        1                   Allison, Master. Hudson Trevor      1  0.9167 Child
3        0                     Allison, Miss. Helen Loraine      1  2.0000 Child
4        0             Allison, Mr. Hudson Joshua Creighton      1 30.0000 Adult
5        0 Allison, Mrs. Hudson J C (Bessie Waldo Daniels)      1 25.0000 Adult
6        1                             Anderson, Mr. Harry      1 48.0000 Adult
  old female sibsp parch alone     fare cherbourg queenstown southampton
1   0 Female     0     0     1 211.3375         0          0           1
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6   0   Male     0     0     1  26.5500         0          0           1
```

- `survived`: 1 (survived) or 0 (not survived)
- `pclass`: passenger class (first, second, third)
- `child`: Adult or Child (under 16 yo)
- `old`: 1 (50+ yo) or 0

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```

- sibsp: number of siblings aboard
- parch: number of parents / children aboard
- fare: Passenger fare (in Pre-1970 British Pounds)
- cherbourg, queenstown, southampton: Embarked at ...

Example: Titanic passenger survival

Let's say we are interested in the following two:

- the effect of fare on survival (i.e., does paying more increase the chance of survival?)
- the effect of `child` and `female` dummies on survival (i.e., was "women and children first" policy implemented?)

	Dependent variable:			
	survived			
	(1)	(2)	(3)	(4)
fare	0.012***	0.012***	0.009***	0.009***
	(0.002)	(0.002)	(0.002)	(0.002)
childChild		0.808***		0.675***
		(0.204)		(0.235)
femaleFemale			2.340***	2.362***
			(0.138)	(0.156)
Constant	-0.882***	-0.893***	-1.718***	-1.722***
	(0.076)	(0.089)	(0.102)	(0.119)
Observations	1,308	1,045	1,308	1,045
Log Likelihood	-827.016	-662.031	-663.249	-528.894
Akaike Inf. Crit.	1,658.032	1,330.061	1,332.498	1,065.788
Note:		*p<0.1;	**p<0.05;	***p<0.01

Interpretation of the coefficients

Log-transformed coefficients

- **glm** in R gives you **log-transformed coefficients** (i.e., log of the odds ratio, or log odds),
- Easy to read the +/- relationship
- Hard to interpret the size of effects purely from the table
- Your interpretation should be: "The coefficient for fare is 0.012, implying a one-unit increase in the ticket fare increases the *log-odds* of survival by 0.47."

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Interpretation of the coefficients

Non-transformed coefficients

- If someone report results from logit using odds ratio (OR) likely due to using STATA, then the result will be different and the interpretation will be different too:
- Hard to read the +/- relationship: OR $>$ 1 (Positive Association), OR $<$ 1 (Negative Association)
- A little easier to interpret the size of effects purely from the table, with a percentage change
- If the odds ratio is 1.25: "A one-unit increase in fare increases the *odds* of survival by 25%."

You can't say anything about *probability* or *likelihood*, even though they are stated in your hypotheses. If you want to interpret it that way, you need to calculate *predicted probability* of your y by varying the values of you x

Model fit

Log likelihood

- **Always negative** (log of "likelihood" = a number between 0 and 1)
- sort of like R^2 (but not really; it doesn't have intuitive interpretation)
- the larger (**smaller in absolute values**), the better

Akaike's Information Criterion (AIC)

- AIC $= -2(L - k)$, where L is the log likelihood and k is the number of coefficients
- sort of like adjusted R^2 (penalizes models with lots of Xs)
- **the smaller, the better**
- Not comparable if n is different

	Dependent variable:			
	survived			
	(1)	(2)	(3)	(4)
fare	0.012***	0.012***	0.009***	0.009***
	(0.002)	(0.002)	(0.002)	(0.002)
childChild		0.808***		0.675***
		(0.204)		(0.235)
femaleFemale			2.380***	2.362***
			(0.155)	(0.156)
Constant	-0.794***	-0.893***	-1.647***	-1.722***
	(0.084)	(0.089)	(0.114)	(0.119)
Observations	1,045	1,045	1,045	1,045
Log Likelihood	-669.970	-662.031	-533.046	-528.894
Akaike Inf. Crit.	1,343.941	1,330.061	1,072.091	1,065.788

Note: *p<0.1; **p<0.05; ***p<0.01

Interpretation

1. Regression table
   - Model (4) is fits the data better based on AICs

2. Fare, child dummy, and female dummy are all positive and significant, as expected

In order to see if the effect of independent variables are also **substantively** significant, we need to show marginal effect. Or more precisely, the predicted probability or the effect plot.

Interpretation: marginal effect

$$\hat{P} = \Lambda(-1.722 + 0.009 * fare + 0.675 * child + 2.362 * female)$$

- Let's first calculate and plot the effect of fare on survival.
- To see the relationship between fare and $\hat{P}$, we calculate $\hat{P}$ for several different values of fare, holding constant other variables at some values.
  - The effect function: `effect(term = "fare", mod = fit.4)` sets everything else constant **at its mean value**.
  - But, mean does not make sense for child / female.
- We should do this for the following four cases:
  - Child, Male
  - Child, Female
  - Adult, Male
  - Adult, Female

Interpretation: marginal effect

```
> # Child, Male
> effect(term = "fare", mod = fit.4,
+   given.values = c(childChild = 1, femaleFemale = 0) )

 fare effect
fare
        0       100       200       300       400       500
0.2598797 0.4703560 0.6919313 0.8503110 0.9349247 0.9732160
```

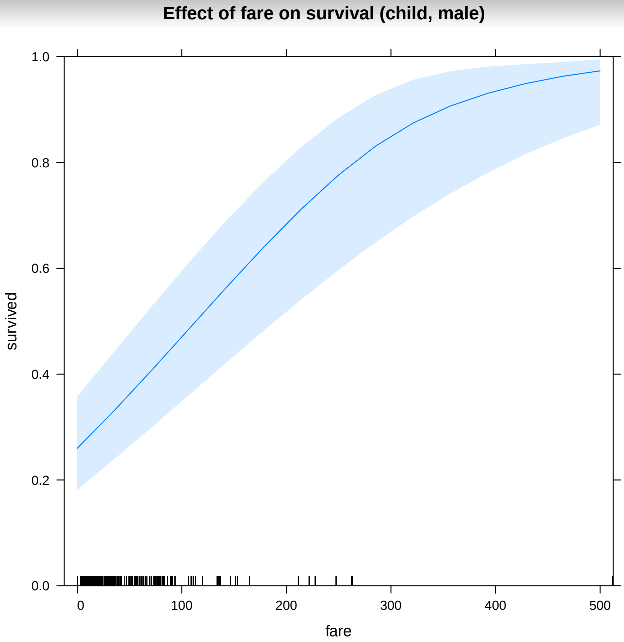
Interpretation: marginal effect

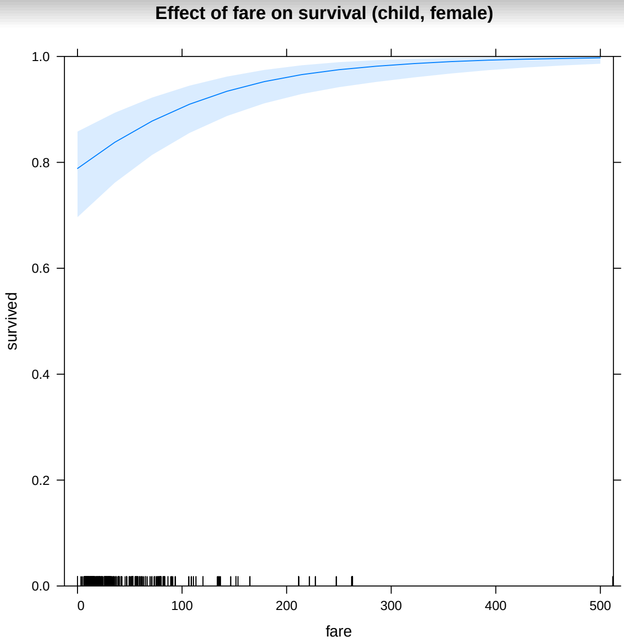
```
> # Child, Male
> effect(term = "fare", mod = fit.4,
+   given.values = c(childChild = 1, femaleFemale = 0) )

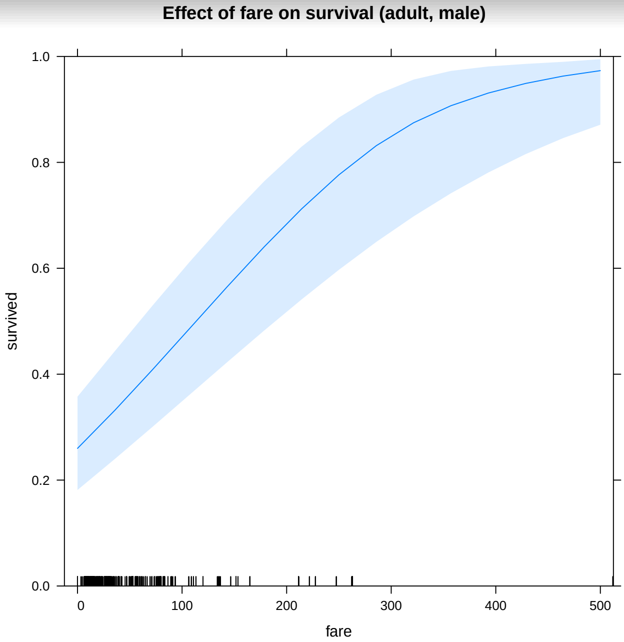
 fare effect
fare
        0       100       200       300       400       500
0.2598797 0.4703560 0.6919313 0.8503110 0.9349247 0.9732160

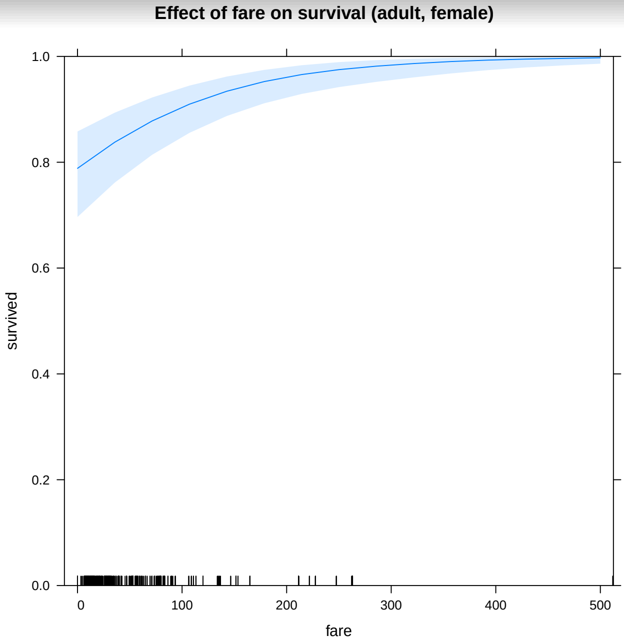
> # Child, Female
> effect(term = "fare", mod = fit.4,
+   given.values = c(childChild = 1, femaleFemale = 1) )

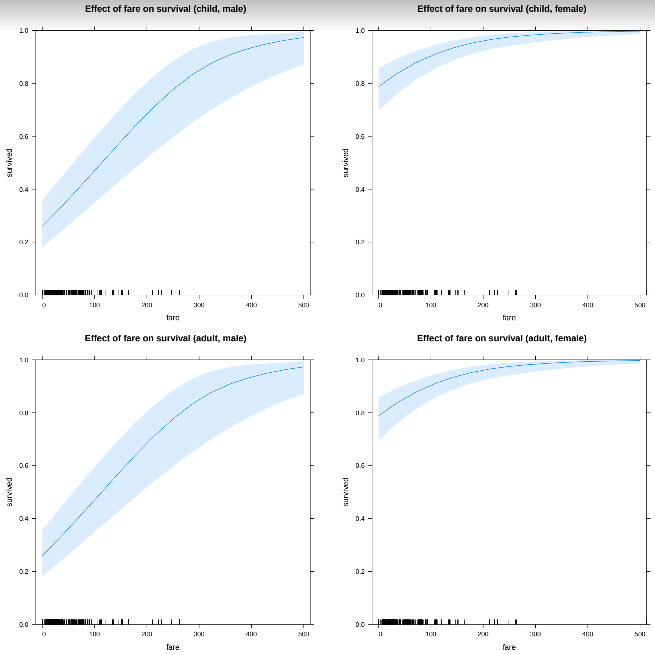
 fare effect
fare
        0       100       200       300       400       500
0.7884491 0.9040867 0.9597422 0.9836853 0.9934850 0.9974139
```


Effect of fare on survival (child, male)
survived
fare
1.0
0.8
0.6
0.4
0.2
0.0
0
100
200
300
400
500


Effect of fare on survival (child, female)
survived
1.0
0.8
0.6
0.4
0.2
0.0
0
100
200
300
400
500
fare


Effect of fare on survival (adult, male)
1.0
0.8
0.6
0.4
0.2
0.0
survived
0
100
200
300
400
500
fare


Effect of fare on survival (adult, female)
survived
fare
1.0
0.8
0.6
0.4
0.2
0.0
0
100
200
300
400
500


Effect of fare on survival (child, male)
Effect of fare on survival (child, female)
Effect of fare on survival (adult, male)
Effect of fare on survival (adult, female)
survived
fare
0.0
0.2
0.4
0.6
0.8
1.0
0
100
200
300
400
500

Interpretation: marginal effect

```
> effect(term = "female", mod = fit.4)

 female effect
female
     Male    Female
0.2129694 0.7417487
```

Interpretation: marginal effect

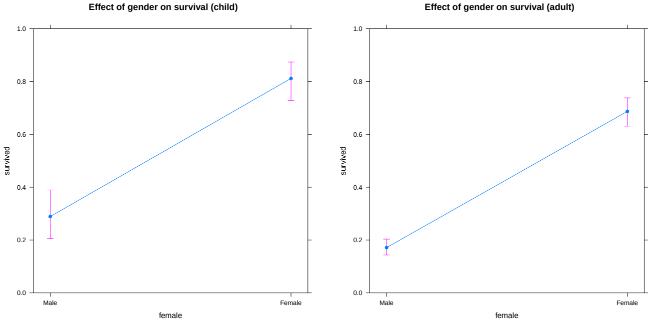
```
> effect(term = "female", mod = fit.4,
+    given.values = c(fare = 15.75, childChild = 1) )

 female effect
female
     Male    Female
0.2889574 0.8117991
```

```
> effect(term = "female", mod = fit.4,
+    given.values = c(fare = 15.75, childChild = 0) )

 female effect
female
     Male    Female
0.1714088 0.6870838
```

Interpretation: marginal effect (gender)



Interpretation: marginal effect

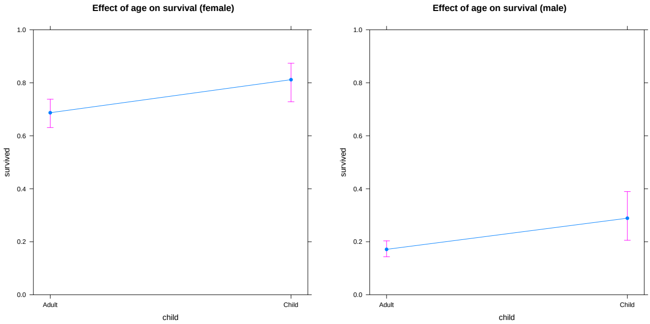
```
> effect(term = "child", mod = fit.4,
+   given.values = c(fare = 15.75, femaleFemale = 1) )

 child effect
child
    Adult     Child
0.6870838 0.8117991

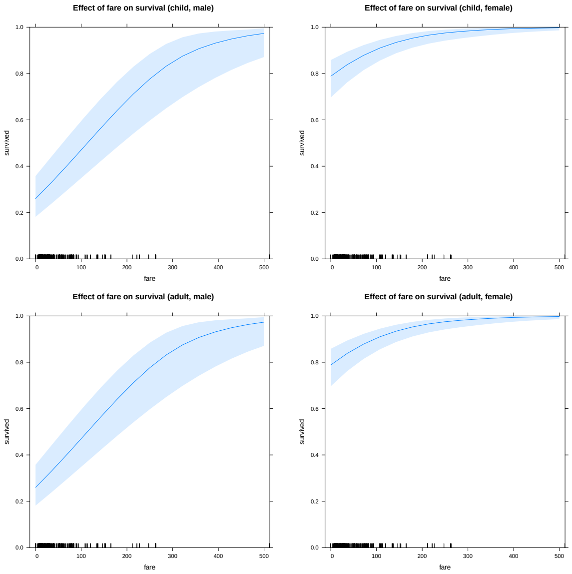
> effect(term = "child", mod = fit.4,
+   given.values = c(fare = 15.75, femaleFemale = 0) )

 child effect
child
    Adult     Child
0.1714088 0.2889574
```

Interpretation: marginal effect (age)



Interpretation: marginal effect (fare)



Summary

Some relationships can only be found by calculating and plotting the marginal effects:

- Passenger's gender has a significant impact on survival probability.
  - For "average" adult passengers (because we held fare at the mean), probability of survival increases from 17% to 69% (= 17% of average adult male passengers survived, whereas 69% of average adult female passengers survived). → *p*.50

Summary

Some relationships can only be found by calculating and plotting the marginal effects:

- Passenger's gender has a significant impact on survival probability.
  - For "average" adult passengers (because we held fare at the mean), probability of survival increases from 17% to 69% (= 17% of average adult male passengers survived, whereas 69% of average adult female passengers survived). → *p*.50
- Passenger's age does have an impact on survival probability, but the effect is much smaller compared with the effect of gender.
  - For "average" male passengers, probability of survival increases from 17% to 29% if he is a child (= 17% of average adult male passengers survived, whereas 29% of average child male passengers survived). → *p*.33
  - For "average" female passenger, probability of survival increases from 69% to 81% if she is a child (= 69% of average adult female passengers survived, whereas 81% of average child female passengers survived).

Homework

Reminder: You should read corresponding Ward & Ahlquist chapters before coming to the class

- Read and summarize: Woo, H. S., Berns, J. P., and Solanelles, P (2023): Write a paragraph summarizing the take away of their proposed strategy for rare event logit.
  ⇒ Woo, H. S., Berns, J. P., & Solanelles, P. (2023). How rare is rare? How common is common? Empirical issues associated with binary dependent variables with rare or common event rates. Organizational Research Methods, 26(4), 655-677.
- Replication and extension