Week 7: Duration Analysis and BTSCS Models

POLI803

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Outline

- Model 1: Duration/survival analysis
    - Key terminology
    - Data structure

- Model 2: Binary-Time-Series-Cross-Sectional analysis

  = how to do duration analysis with a logit model

Duration analysis

Duration analysis (econ) = survival analysis (health science) = event history analysis (stats)

Types of questions we ask:

- Logit analysis (DV = binary):
  - DV = event occurring or not occurring
  - Does X make it more likely for an event to occur?
- **Duration analysis (DV = time):**
  - DV = time until event occurring (e.g., war, arrest)
  - Does X prolong the duration of time until the unit experiences the event?

Failure time process

Duration data are generated by a failure time process:

- Units: individuals, governments, countries
- Units are initially in some state: healthy, democracy, at peace
- At any given point in time, units are **"at risk"** of experiencing some event (failure):
  - individual die
  - governments may become autocratic
  - countries go to war
- Event (failure) = transition from one state to another state

Failure time process

If a unit experiences an event (failure), then we observe the duration until the event

- DV (survival time; failure time) = duration until the event
- = how long a unit survives until it experiences a failure event
- Time units can be measured in **years, months, days, hours, seconds, etc.**

Failure time process

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    - Some countries may never go to war (survive forever)

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    - Let's say the failure event of interest is "die from a lung cancer"
    - When a person is killed in a traffic accident, s/he will not die from a lung cancer

Failure time process

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    - Let's say the failure event of interest is "die from a lung cancer"
    - When a person is killed in a traffic accident, s/he will not die from a lung cancer

- When a unit does not experience a failure event, then we **cannot observe** the full duration until the event
  - We call these units **"censored"** observations
  - Censored units are still informative, as we can still partially observe the duration of survival

Examples of duration data

Duration of democratic regimes

- Unit: democratic country
- Unit of time: year
- Initial state: democracy
- Failure event: Autocratic reversal (breakdown of democracy)
- If democracy never fails in a country, that country is censored
- Censoring indicator: 1 if eventually failed, 0 if censored
- DV = duration until democracy fails, or duration until the end of observation period (1700–2001)

Examples of duration data: Failed Democracy

Country	Begin	End	Time	Failed?
Grenada	1974	1979	6	Yes
Cuba	1909	1925	17	Yes
Cuba	1940	1952	13	Yes
United States	1789	2001	213	No
Canada	1867	2001	135	No
⋮	⋮	⋮	⋮	⋮

- Democracy broke down in Grenada (in 1979) and in Cuba (in 1925 and again in 1952)
- Observations are censored for US and Canada (never failed)

Examples of duration data: Peace Duration

Duration of peace

- Unit: country
- Unit of time: year
- Initial state: peace
- "Failure" event: war onset
- DV = duration of peace / survival of peace
- If war never happens in a country by the end of the observation period, that country is censored

Examples of duration data: Transition of Power

Duration of cabinet

- Unit: cabinet in parliamentary democracies
- Initial state: in power
- Failure event: dissolution or election
- If a cabinet has not failed by the end of the observation period, the cabinet is censored
- Time = duration until cabinet ends due to dissolution or election, or duration until the end of observation period

Why not linear regression?

Although duration is an interval-level (continuous) variable, running linear regression is not appropriate

- Negative predicted values don't make sense
- Censoring

Models for duration analysis 1

Continuous-time duration models (survival models; event-history models)

- Parametric: Exponential, Weibull, Log-logistic, Log-normal, Generalized Gamma, etc.
  - makes big assumptions on the data generating processes
  - efficient

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- Semi-parametric: **Cox model**
    - makes no assumptions on the data generating processes
    - makes a specific parametric assumption about the hazard ratios of different explanatory variables but does not assume any particular parametric form for the baseline hazard function over time
    - flexible

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  - makes a specific parametric assumption about the hazard ratios of different explanatory variables but does not assume any particular parametric form for the baseline hazard function over time
  - flexible
- Survival analysis can be a standalone course
- We will only see how to interpret the results (but not how to estimate them)

Models for duration analysis 1

Three ways to report the estimated results (usually made explicit in a table footnote)

- If results are reported in AFT (Accelerated Failure Time) metric: positive coefficients $\rightsquigarrow$ longer duration
- If results are reported in "Hazard Rate": positive coefficients $\rightsquigarrow$ greater risk $\rightsquigarrow$ shorter duration
- If results are reported in "Hazard Ratio": coefficients are all positive. Coefficients greater than 1 $\rightsquigarrow$ greater risk $\rightsquigarrow$ shorter duration

Chiba et al. (2015): AFT

Table 3 Competing risks analysis of government survival: models without selection versus models with selection

Explanatory variables	*Replacement terminations*		*Dissolution terminations*	
	Without selection	*With selection*	*Without selection*	*With selection*
Minority government	−0.271***	−0.201**	−0.362***	−0.325**
	(0.091)	(0.091)	(0.137)	(0.139)
Ideological divisions in coalition	−0.005***	−0.002	0.003	0.005
	(0.002)	(0.002)	(0.004)	(0.004)
Returnability	−0.201**	−0.359***	−0.015	−0.078
	(0.100)	(0.111)	(0.140)	(0.150)
Effective number of legislative parties	−0.063**	−0.006	0.074	0.107
	(0.031)	(0.035)	(0.058)	(0.067)
Polarization index	−0.032*	−0.022	−0.066**	−0.064**
	(0.020)	(0.020)	(0.027)	(0.028)
Time remaining in CIEP (Logged)	0.894***	0.895***	0.752***	0.753***
	(0.065)	(0.066)	(0.117)	(0.151)
Intercept	1.304***	1.334***	2.058**	2.018*
	(0.494)	(0.505)	(0.891)	(1.155)
Duration dependence (Logged)	0.540***	0.683***	0.488***	0.543***
	(0.057)	(0.060)	(0.082)	(0.092)
Error correlation ($\tanh^{-1}(\theta)$)		0.310***		0.112
		(0.073)		(0.093)
Log-likelihood	−2655.72	−2646.22	−1803.16	−1802.42

Note: Cell entries are coefficient estimates (with standard errors in parentheses) expressed in the accelerated failure-time metric. All models assume a Weibull parameterization of the baseline hazard rate. Total number of government terminations: 432. Number of terminations resulting in nonelectoral replacement: 231. Number of terminations resulting in early elections: 112. Number of potential governments in selection models: 95,576 (output from selection component of models with selection shown in Appendix Table 1 in the Supplementary Materials for this article). Significance levels: *: 10%; **: 5%; ***: 1%.

Positive coefficients $\rightsquigarrow$ longer duration

Gibler & Tir (2010): Hazard Rate

TABLE 3 Cox Regressions of State-Level Democratization Following a Peaceful Territorial Transfer, 1945–2000

	Number of Borders Adjusted by Transfer
Peaceful Transfer	0.443(0.185)*
Number of Borders	−0.068(0.077)
Nonterritorial MIDs	−0.284(0.418)
Territorial MIDs	−0.969(0.761)
Economic Development	−0.330(0.082)**
Regime Score	0.028(0.028)
% Democracies in Region	2.237(0.884)*
(ln) Capabilities	0.102(0.105)
N	4,662
Chi-square	29.24**

Note: The Peaceful Transfer variable indicates whether a state's borders have been adjusted peacefully. Cell entries report Cox coefficients and robust standard errors (in parentheses). The unit of analysis is a country-year. All independent variables are lagged with respect to the dependent variable. Observations that were already democratic prior to the transfer have been dropped. Significance levels are one-tailed: *p < .05; **p < .01.

Positive coefficients ⇝ greater risk ⇝ shorter duration (quicker transition to the event)

Cunningham (2011): Hazard Ratio

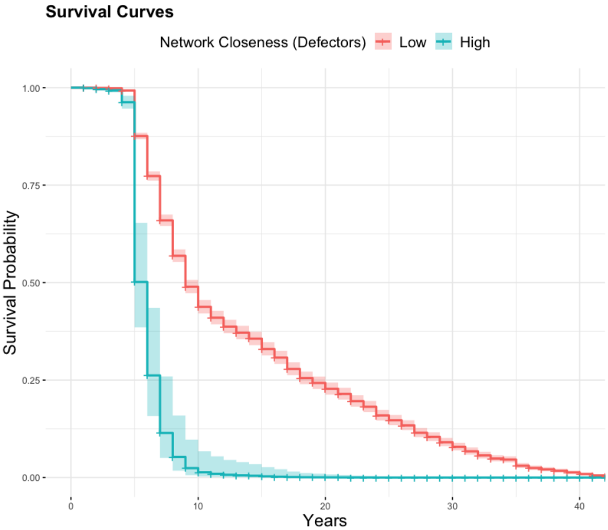
TABLE 5. Hazard Ratios[a]

	Model 1 Violence	Model 2 Violence	Model 3 New Concessions	Model 4 New Concessions
Unitary movement	0.19*		0.39**	
	(0.18)		(0.15)	
Number of SD factions (log)		2.54**		0.97
		(0.78)		(0.23)
Relative size of group	1.06*	1.07*	1.00	1.01
	(0.03)	(0.04)	(0.04)	(0.04)
Territorial base	0.43	0.39	0.28**	0.39**
	(.31)	(0.28)	(0.09)	(0.13)
State population (log)	1.41*	1.41*	1.69**	1.45**
	(.30)	(0.28)	(0.35)	(0.26)
GDP per capita (log)	0.60**	0.62**	2.05**	2.06*
	(0.13)	(0.15)	(0.76)	(0.78)
Military expenditure per capita	1.00**	1.00	0.999*	0.999*
	(0.00)	(0.00)	(0.00)	(0.00)
Number of subjects	87	87	87	87
Number of failures	18	18	40	40
Time at risk	526	526	526	526
Log pseudo likelihood	−48.88	−48.04	−71.18	−72.91

Note: GDP, gross domestic product.
[a] A hazard ratio less than 1 indicates that failure is less likely at any given point in time; greater than 1 indicates failure is more likely to happen.
* Statistically significant at the 0.10 level; ** statistically significant at the 0.05 level in two-tailed tests.

Coefficients greater than 1 ⇝ greater risk ⇝ shorter duration (quicker transition to the event)

Liu (2022): Cox model. The effect of networks on remaining uncaptured (survival)



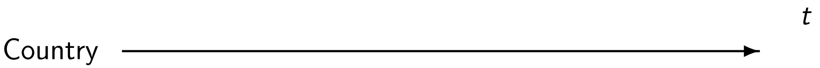
Models for duration analysis 2

Discrete-time duration models, a.k.a, **Binary Time-Series Cross-Section (BTSCS) models**

- Some empirical research still adopts this approach, because we can account for temporal dependence, which Cox model completely ignores
- But the majority of research nowadays uses the Cox model
- With some tricks, we can convert duration data into BTSCS data
  - Duration data $\leftrightarrows$ BTSCS data
- We apply logit / probit models to the BTSCS data

Duration data $\leftrightarrow$ BTSCS Data

(1) Continuous time duration data



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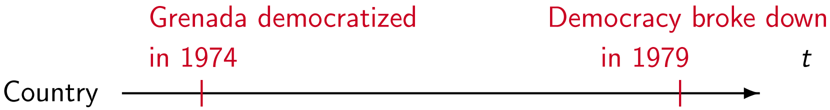

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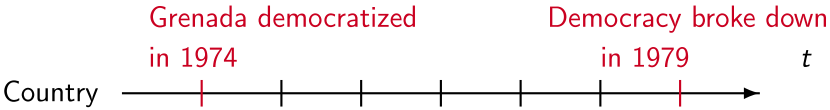

DV is coded as 6 years

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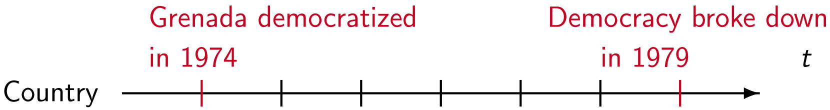

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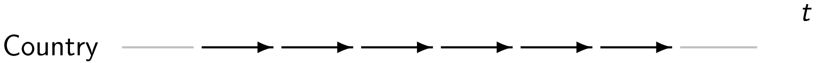
(2) BTSCS Data (discrete time duration data)



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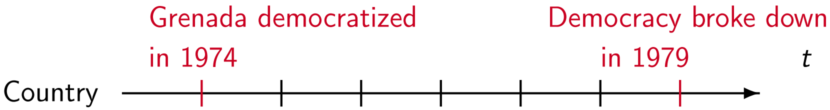

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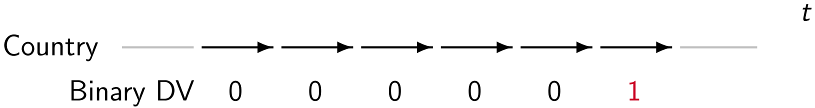

Data structure

Table: Continuous time

Country	Begin	End	Time	Failed?
⋮	⋮	⋮	⋮	⋮
Grenada	1974	1979	6	Yes
Canada	1867	2001	135	No
⋮	⋮	⋮	⋮	⋮

Data structure

Table: Continuous time

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⋮	⋮	⋮	⋮	⋮
Grenada	1974	1979	6	Yes
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⋮	⋮	⋮	⋮	⋮

Table: BTSCS

Unit	Year	Event
Grenada	1974	0
Grenada	1975	0
Grenada	1976	0
Grenada	1977	0
Grenada	1978	0
Grenada	1979	1
⋮	⋮	⋮
Canada	1867	0
Canada	1868	0
⋮	⋮	⋮
Canada	2000	0
Canada	2001	0
⋮	⋮	⋮

BTSCS Estimation

Unit	Year	Event
Grenada	1974	0
Grenada	1975	0
Grenada	1976	0
Grenada	1977	0
Grenada	1978	0
Grenada	1979	1
Canada	1867	0
Canada	1868	0
⋮	⋮	⋮
Canada	2000	0
Canada	2001	0

Let's say we are interested in the effect of Military (whether or not a country has a standing military forces in a given year) on democratic survival

BTSCS Estimation

Unit	Year	Y: Event	X: Military?
Grenada	1974	0	No
Grenada	1975	0	No
Grenada	1976	0	No
Grenada	1977	0	No
Grenada	1978	0	No
Grenada	1979	1	No
Canada	1867	0	Yes
Canada	1868	0	Yes
⋮	⋮	⋮	⋮
Canada	2000	0	Yes
Canada	2001	0	Yes

BTSCS Estimation

Unit	Year	Y: Event	X: Military?
Grenada	1974	0	No
Grenada	1975	0	No
Grenada	1976	0	No
Grenada	1977	0	No
Grenada	1978	0	No
Grenada	1979	1	No
Canada	1867	0	Yes
Canada	1868	0	Yes
⋮	⋮	⋮	⋮
Canada	2000	0	Yes
Canada	2001	0	Yes

$$Y^* = \alpha + \beta * Military$$

$$\hat{P} = \Lambda(Y^*)$$

BTSCS Estimation

Unit	Year	Y: Event	X: Military?
Grenada	1974	0	No
Grenada	1975	0	No
Grenada	1976	0	No
Grenada	1977	0	No
Grenada	1978	0	No
Grenada	1979	1	No
Canada	1867	0	Yes
Canada	1868	0	Yes
⋮	⋮	⋮	⋮
Canada	2000	0	Yes
Canada	2001	0	Yes

$$Y^* = \alpha + \beta * Military$$

$$\hat{P} = \Lambda(Y^*)$$

BTSCS Estimation

However, the predicted probabilities of an event may well depend on time

$$\Pr(\text{event} \mid 1 \text{ year after democratization}) \text{ may not be the same as}$$
$$\Pr(\text{event} \mid 2 \text{ years after democratization}) \text{ or}$$
$$\Pr(\text{event} \mid 3 \text{ years after democratization}) \text{ or}$$
$$\vdots$$
$$\Pr(\text{event} \mid n \text{ years after democratization})$$

- What is the potential issue here?

BTSCS Estimation

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$$\begin{aligned}
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&\Pr(\text{event} \mid n \text{ years after democratization})
\end{aligned}$$

- What is the potential issue here? → time dependence

BTSCS Estimation

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$$\vdots$$
$$\Pr(\text{event} \mid n \text{ years after democratization})$$

- What is the potential issue here? → time dependence
- The previous model imposes a structure where all of them are the same; But in fact, **temporal dependency** in data is obvious

BTSCS Estimation

Unit	Year	Y: Event	X: Military?
Grenada	1974	0	No
Grenada	1975	0	No
Grenada	1976	0	No
Grenada	1977	0	No
Grenada	1978	0	No
Grenada	1979	1	No
Canada	1867	0	Yes
Canada	1868	0	Yes
⋮	⋮	⋮	⋮
Canada	2000	0	Yes
Canada	2001	0	Yes

What can we do to allow $\hat{P}$ to be different depending on time since the starting year?

BTSCS Estimation

Unit	Year	Y: Event	X: Military?	Time Counter
Grenada	1974	0	No	0
Grenada	1975	0	No	1
Grenada	1976	0	No	2
Grenada	1977	0	No	3
Grenada	1978	0	No	4
Grenada	1979	1	No	5
Canada	1867	0	Yes	0
Canada	1868	0	Yes	1
⋮	⋮	⋮	⋮	⋮
Canada	2000	0	Yes	133
Canada	2001	0	Yes	134

BTSCS Estimation

Unit	Year	Y: Event	X: Military?	Time Counter
Grenada	1974	0	No	0
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Grenada	1976	0	No	2
Grenada	1977	0	No	3
Grenada	1978	0	No	4
Grenada	1979	1	No	5
Canada	1867	0	Yes	0
Canada	1868	0	Yes	1
⋮	⋮	⋮	⋮	⋮
Canada	2000	0	Yes	133
Canada	2001	0	Yes	134

$$Y^* = \alpha + \beta_1 * Military + \beta_2 * Time\ Counter$$

$$\hat{P} = \Lambda(Y^*)$$

BTSCS Estimation

$$Y^* = \alpha + \beta_1 * Military + \beta_2 * Counter$$
$$\hat{P} = \Lambda(Y^*)$$

- What's the issue here?

BTSCS Estimation

$$Y^* = \alpha + \beta_1 * Military + \beta_2 * Counter$$
$$\hat{P} = \Lambda(Y^*)$$

- What's the issue here?
- What does a negative / positive β_2 imply?

BTSCS Estimation

$$Y^* = \alpha + \beta_1 * \mathit{Military} + \beta_2 * \mathit{Counter}$$
$$\hat{P} = \Lambda(Y^*)$$

- What's the issue here?
- What does a negative / positive β_2 imply?
- One big drawback of the model above is that it assumes monotonic relationship between $\hat{P}$ and time

BTSCS Estimation

The following is more flexible, as it allows for quadratic (U shape or inverse-U shape)

$$Y^* = \alpha + \beta_1 * Military + \beta_2 * Counter + \beta_3 * Counter^2$$
$$\hat{P} = \Lambda(Y^*)$$

Carter & Signorino (2010) showed that cubic model is usually enough

$$Y^* = \alpha + \beta_1 * Military + \beta_2 * Counter + \beta_3 * Counter^2 + \beta_4 * Counter^3$$
$$\hat{P} = \Lambda(Y^*)$$

BTSCS Estimation

- The "Counter" variable sometimes called spell, t, time, or "time since last event"
- In conflict research it's often called peace years
- Sometimes people use $log(t+1)$ or $\sqrt{t}$ instead of cubic polynomial
- Before Carter & Signorino (2010), "splines" used to be commonly used (but not any more)

Fake data example

Let's say we have the following continuous-time data

Table: Continuous Time

Unit	Begin	End	Time	Failed?	X
A	1974	1979	6	Yes	1
B	1990	1991	2	Yes	0
C	1995	2001	7	No	1
D	1992	2000	9	Yes	1
E	1970	1972	3	Yes	0
F	1969	1975	7	Yes	0

We will see how to convert this into a BTSCS data set, how to estimate BTSCS models, and how to do model selection

Fake data example: steps

1. Expand the data set (i.e., create duplicates)
   To do so, we use the `untable` function from the `reshape` package
2. Assign observation ID (a sequence of numbers from 1 to n per unit where n is the total number of observations in each unit)
3. Create a binary DV that is equal to 1 if and only if
   - ID is equal to Time (i.e., if the observation is the last one per unit)
   - Failed is Yes (i.e., if it's not censored)
4. Create a calendar variable
5. Create a counter variable using the `btscs` function from the `DAMisc` package

BTSCS estimation

Once you obtained the BTSCS data set, try estimating at least the following logit models

- A model without any time component
- A model with the counter variable, t (linear time model)
- A model with t and t^2 (quadratic polynomial model)
- A model with t, t^2, and t^3 (cubic polynomial model)
- A model with $log(t+1)$
- A model with $\sqrt{t}$

then choose the one that yields the smallest AIC

Do NOT choose one model over another based on the statistical significance of your favorite independent variable(s)

Note on given.values

When using quadratic or cubic polynomials, be extra careful in calculating the substantive effects of other variables

- You should NOT set the values of t, t^2, and t^3 at their mean values
- When you set t at mean(t), t^2 should be set equal to mean$(t)^2$, not mean(t^2)
- (mean $t)^2 \neq$ mean of t^2
- This applies to variables other than time

Note on interpretation

Be careful in interpreting the signs of the coefficients

- In continuous time duration models, the interpretation depends on representation
    - AFT: positive coefficient = longer duration = smaller risks
    - Hazard rate: positive coefficient = shorter duration = larger risks
    - Hazard ratio: coefficient > 1 = shorter duration = larger risks
- In BTSCS models: positive coefficients = greater risk of a failure event = shorter duration
- Always look at the substantive/marginal effect plots!!

References

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