Week 5: Multinomial Logit Model

POLI803

Howard Liu

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University of South Carolina

Review: Latent Variable Approach

Y^*: unobservable utility of taking certain actions

$$Y^* = \boldsymbol{X}\beta + \epsilon$$

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Binary outcome (logit)

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Binary outcome (logit)

$$Y = \begin{cases} 1 & \text{if } Y^* > 0 \\ 0 & \text{if } Y^* \leq 0 \end{cases}$$

Ordered outcome (ordered logit)

$$Y = \begin{cases} 2 & \text{if } Y^* > cut_2 \\ 1 & \text{if } cut_1 < Y^* \leq cut_2 \\ 0 & \text{if } Y^* \leq cut_1 \end{cases}$$

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  for A, B, C, we have Y_A^*, Y_B^*, and Y_C^*

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$$Y = \begin{cases} A & \text{if } Y_A^* > Y_B^* \text{ and } Y_A^* > Y_C^* \\ B & \text{if } Y_B^* > Y_A^* \text{ and } Y_B^* > Y_C^* \\ C & \text{if } Y_C^* > Y_A^* \text{ and } Y_C^* > Y_B^* \end{cases}$$

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- Things get more complicated when we have an **unordered categorical** DV
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- The model is called **multinomial logit model**

Multinomial Logit Model

When we have k number of categories, we assume k number of utilities:

$$\begin{aligned} Y_A^* &= \boldsymbol{X}\beta_{\boldsymbol{A}} + \epsilon_A \\ Y_B^* &= \boldsymbol{X}\beta_{\boldsymbol{B}} + \epsilon_B \\ Y_C^* &= \boldsymbol{X}\beta_{\boldsymbol{C}} + \epsilon_C \end{aligned}$$

$\boldsymbol{X}$ is common, but β differs

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$\boldsymbol{X}$ is common, but β differs

We set one category as the baseline, estimating $k-1$ sets of $\boldsymbol{\beta}$s

$$Y_A^* = \boldsymbol{X}\beta_{\boldsymbol{A}} + \epsilon_A$$
$$Y_B^* = \boldsymbol{X}\beta_{\boldsymbol{B}} + \epsilon_B$$
$$Y_C^* = 0$$

Multinomial Logit Model

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When we use C as the baseline

- the estimated $\beta_{\boldsymbol{A}}$ shows the effect of $\boldsymbol{X}$ on the utility of choosing A **relative to** C
- the estimated $\beta_{\boldsymbol{B}}$ shows the effect of $\boldsymbol{X}$ on the utility of choosing B **relative to** C

Multinomial Logit Model

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Multinomial Logit Model

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Multinomial Logit Model

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Multinomial Logit Model

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Multinomial Logit Model

$$Y_A^* = \boldsymbol{X}\boldsymbol{\beta_A} + \epsilon_A$$
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When we use C as the baseline

- the estimated $\boldsymbol{\beta_A}$ shows the effect of $\boldsymbol{X}$ on the utility of choosing A **relative to** C
- the estimated $\boldsymbol{\beta_B}$ shows the effect of $\boldsymbol{X}$ on the utility of choosing B **relative to** C
- contrarily, the effect of $\boldsymbol{X}$ on the utility of choosing C relative to A is $-\boldsymbol{\beta_A}$
- contrarily, the effect of $\boldsymbol{X}$ on the utility of choosing C relative to B is $-\boldsymbol{\beta_B}$
- the effect of $\boldsymbol{X}$ on the utility of choosing A relative to B is ?

Multinomial Logit Model

When we use C as the baseline, the effect of $\boldsymbol{X}$ on the utility of choosing A relative to B is not shown

Multinomial Logit Model

When we use C as the baseline, the effect of $\boldsymbol{X}$ on the utility of choosing A relative to B is not shown

We thus have to estimate an alternative model as well:

$$
\begin{aligned}
Y_A^* &= \boldsymbol{X}\beta'_{\boldsymbol{A}} + \epsilon_A \\
Y_B^* &= 0 \\
Y_C^* &= \boldsymbol{X}\beta'_{\boldsymbol{C}} + \epsilon_C
\end{aligned}
$$

- the estimated $\beta'_{\boldsymbol{A}}$ shows the effect of $\boldsymbol{X}$ on the utility of choosing A relative to B

Multinomial Logit Model

When we use C as the baseline, the effect of $\boldsymbol{X}$ on the utility of choosing A relative to B is not shown

We thus have to estimate an alternative model as well:

$$Y_A^* = \boldsymbol{X}\boldsymbol{\beta'_A} + \epsilon_A$$
$$Y_B^* = 0$$
$$Y_C^* = \boldsymbol{X}\boldsymbol{\beta'_C} + \epsilon_C$$

- the estimated $\boldsymbol{\beta'_A}$ shows the effect of $\boldsymbol{X}$ on the utility of choosing A relative to B
- the estimated $\boldsymbol{\beta'_C}$ shows the effect of $\boldsymbol{X}$ on the utility of choosing C relative to B, which is equal to $-\boldsymbol{\beta_B}$ from the model where C is the baseline

Multinomial Logit Model

Therefore, when we have three outcomes: A, B, and C, we report three sets of results:

1. A vs B (the effect of $\boldsymbol{X}$ on Y_A^* relative to Y_B^*)
2. A vs C (the effect of $\boldsymbol{X}$ on Y_A^* relative to Y_C^*)
3. B vs C (the effect of $\boldsymbol{X}$ on Y_B^* relative to Y_C^*)

A combination of choosing 2 from k categories C_2^k

Multinomial Logit Model

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A combination of choosing 2 from k categories C_2^k

- To obtain (2) and (3), we estimate a model using C as the baseline
- To obtain (1), we estimate a model using B as the baseline

Example: Territorial Disputes

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  Three unordered outcomes can be distinguished

  - Most of the time they do nothing: **Status quo (SQ)**
  - Sometimes states engage in a negotiation: **Negotiation**
  - Sometimes states fight over the territory: **Militarization**

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- Disputes over territory are particularly war-prone

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  Three unordered outcomes can be distinguished

  - Most of the time they do nothing: **Status quo (SQ)**
  - Sometimes states engage in a negotiation: **Negotiation**
  - Sometimes states fight over the territory: **Militarization**

- Data (Paul Huth and his collaborators)

  - Unit: dispute-month, 1944 – 2000
  - Outcome: SQ, Negotiation, Militarization

Example: Territorial Disputes

Huth, Crocco, and Appel (2012, *ISQ*)

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Hypotheses:

- When challenger has a legal advantage, SQ becomes less likely (than other outcomes)
- When challenger has a legal advantage, Negotiation becomes more likely
- When challenger has a legal advantage, Militarization becomes less likely

Distribution of DV

DV:

```
> table(td $ dvsqb)

   0    1    2
2459 1140  241
```

Distribution of DV

DV:

```
> td $ dvsqb.cat <- factor(td $ dvsqb,
+           label = c("SQ","Neg", "Mil"))
> table(td $ dvsqb.cat)

  SQ  Neg  Mil
2459 1140  241
```

Distribution of IDV

IDV:

```
> table(td $ slc3b)

   0    1
3671  169
```

Distribution of IDV

IDV: List of disputants where IDV = 1

	challenger	target
7	Botswana	Namibia
532	Chad	Libyan Arab Jamahiriya
745	Egypt	Israel
809	Iran, Islamic Republic Of	United Kingdom
1610	Argentina	Uruguay
2037	Nicaragua	United States
2184	Paraguay	Argentina
2321	Afghanistan	Russian Federation
3298	Portugal	Indonesia
3451	Cyprus	Turkey
3477	Czechoslovakia	Hungary
3519	France	Italy
3683	Romania	Hungary
3791	German Federal Republic	France

Multinomial Logit: Estimation

To estimate a multinomial logit model, we use the `multinom` function from the `nnet` package

```
> library(nnet)
> fit.0 <- multinom(dvsqb.cat ~ slc3b, data = td)
# weights:  9 (4 variable)
initial  value 4218.671188
iter  10 value 3137.766235
final  value 3137.761465
converged
```

In fitting a multinomial logit model, `R` assumes that the first category (in our case, SQ) is the baseline

Multinomial Logit: Results

	Dependent variable:	
	Neg vs. SQ	Mil vs. SQ
	(1)	(2)
slc3b	0.717***	−0.030
	(0.162)	(0.376)
Constant	−0.804***	−2.322***
	(0.037)	(0.069)
Akaike Inf. Crit.	6,283.523	6,283.523
Note:	*p<0.1; **p<0.05; ***p<0.01	

- Results from one model shown in two columns

Multinomial Logit: Results

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- Results from one model shown in two columns
- SQ is the baseline
  - 0.717 is the effect of `slc3b` on Y^*_{Neg} relative to Y^*_{SQ}
  - -0.030 is the effect of `slc3b` on Y^*_{Mil} relative to Y^*_{SQ}

Multinomial Logit: Results

We have obtained two sets of coefficients so far:

- Y^*_{Neg} vs Y^*_{SQ}
- Y^*_{Mil} vs Y^*_{SQ}

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- Y^*_{Neg} vs Y^*_{SQ}
- Y^*_{Mil} vs Y^*_{SQ}

However, we haven't got coefficients for:

- Y^*_{Neg} vs Y^*_{Mil}

Multinomial Logit: Results

We have obtained two sets of coefficients so far:

- Y^*_{Neg} vs Y^*_{SQ}
- Y^*_{Mil} vs Y^*_{SQ}

However, we haven't got coefficients for:

- Y^*_{Neg} vs Y^*_{Mil}

We have to do another round of estimation (of the same model), using either `Neg` or `Mil` as the baseline

Multinomial Logit: Recoding the DV

Note that R automatically chooses the first category as the baseline:

$$\texttt{dvsqb} = \begin{cases} 0 & \text{for SQ} \\ 1 & \text{for Neg} \\ 2 & \text{for Mil} \end{cases}$$

We create a new variable that assigns a different number to the category, so we got the order we want

$$\texttt{dvsqb.base1} = \begin{cases} 1 & \text{for Neg} \\ 2 & \text{for Mil} \\ 3 & \text{for SQ} \end{cases}$$

Multinomial Logit: Results

	Dependent variable:	
	Mil vs. Neg	SQ vs. Neg
	(1)	(2)
slc3b	-0.747^{**}	-0.717^{***}
	(0.378)	(0.162)
Constant	-1.518^{***}	0.804^{***}
	(0.072)	(0.037)
Akaike Inf. Crit.	6,283.523	6,283.523
Note:	*p$<$0.1; **p$<$0.05; ***p$<$0.01	

- Results from one model shown in two columns
- Neg is the baseline
  - -0.747 is the effect of `slc3b` on Y^*_{Mil} relative to Y^*_{Neg}
  - -0.717 is the effect of `slc3b` on Y^*_{SQ} relative to Y^*_{Neg}

Multinomial Logit: Combined Results

	Dependent variable:			
	Neg vs. SQ	Mil vs. SQ	Mil vs. Neg	SQ vs. Neg
	(1)	(2)	(3)	(4)
slc3b	0.717***	−0.030	−0.747**	−0.717***
	(0.162)	(0.376)	(0.378)	(0.162)
Constant	−0.804***	−2.322***	−1.518***	0.804***
	(0.037)	(0.069)	(0.072)	(0.037)
Akaike Inf. Crit.	6,283.523	6,283.523	6,283.523	6,283.523
Note:			*p<0.1; **p<0.05; ***p<0.01	

- SQ is the baseline in (1) and (2)
- Neg is the baseline in (3) and (4)
- $\beta_{(4)}$ is equal to $-\beta_{(1)}$

Multinomial Logit: Combined Results

	Dependent variable:		
	Neg vs SQ	Mil vs SQ	Mil vs Neg
	(1)	(2)	(3)
slc3b	0.717^{***}	-0.030	-0.747^{**}
	(0.162)	(0.376)	(0.378)
Constant	-0.804^{***}	-2.322^{***}	-1.518^{***}
	(0.037)	(0.069)	(0.072)
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Note:			*p<0.1; **p<0.05; ***p<0.01

Multinomial Logit: Combined Results

	Dependent variable:		
	Neg vs SQ	Mil vs SQ	Mil vs Neg
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Constant	−0.804***	−2.322***	−1.518***
	(0.037)	(0.069)	(0.072)
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Note:			*p<0.1; **p<0.05; ***p<0.01

- Legal advantage makes Neg more likely relative to SQ

Multinomial Logit: Combined Results

	Dependent variable:		
	Neg vs SQ	Mil vs SQ	Mil vs Neg
	(1)	(2)	(3)
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	(0.162)	(0.376)	(0.378)
Constant	−0.804***	−2.322***	−1.518***
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Note:			*p<0.1; **p<0.05; ***p<0.01

- Legal advantage makes Neg more likely relative to SQ
- Legal advantage makes Neg more likely relative to Mil

Multinomial Logit: Combined Results

	Dependent variable:		
	Neg vs SQ	Mil vs SQ	Mil vs Neg
	(1)	(2)	(3)
slc3b	0.717***	−0.030	−0.747**
	(0.162)	(0.376)	(0.378)
Constant	−0.804***	−2.322***	−1.518***
	(0.037)	(0.069)	(0.072)
Akaike Inf. Crit.	6,283.523		
Note:			*p<0.1; **p<0.05; ***p<0.01

- Legal advantage makes Neg more likely relative to SQ
- Legal advantage makes Neg more likely relative to Mil
- Legal advantage doesn't make Mil more/less likely relative to SQ

Multinomial Logit: Substantive Effects

- Recall that coefficients (β) represents the effect of X on Y^*, but we are interested in their effects on outcome probabilities

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- We use the `effect` function to calculate substantive effects (marginal effects)

Multinomial Logit: Substantive Effects

- Recall that coefficients (β) represents the effect of X on Y^*, but we are interested in their effects on outcome probabilities
- We use the `effect` function to calculate substantive effects (marginal effects)
- Note: we have done two sets of estimation of the same model

  No matter which version we use, we will get the same substantive effects

Multinomial Logit: Substantive Effects

```
> effect(term = "slc3b", mod = fit.0)

slc3b effect (probability) for SQ
slc3b
        0       0.2       0.4       0.6       0.8         1
0.6469627 0.6195382 0.5906089 0.5603775 0.5290925 0.4970415

slc3b effect (probability) for Neg
slc3b
        0       0.2       0.4       0.6       0.8         1
0.2895669 0.3200413 0.3521326 0.3856162 0.4202179 0.4556212

slc3b effect (probability) for Mil
slc3b
         0        0.2        0.4        0.6        0.8          1
0.06347044 0.06042046 0.05725845 0.05400624 0.05068957 0.04733728
```

Multinomial Logit: Substantive Effects

```
> effect(term = "slc3b", mod = fit.0)

slc3b effect (probability) for SQ
slc3b
        0       0.2       0.4       0.6       0.8         1
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slc3b effect (probability) for Neg
slc3b
        0       0.2       0.4       0.6       0.8         1
0.2895669 0.3200413 0.3521326 0.3856162 0.4202179 0.4556212

slc3b effect (probability) for Mil
slc3b
         0        0.2        0.4        0.6        0.8          1
0.06347044 0.06042046 0.05725845 0.05400624 0.05068957 0.04733728
```

There's something wrong here. What is that?

Multinomial Logit: Substantive Effects

- The IDV we have is a binary variable (can only take 0 OR 1)

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- We should only have two sets of probabilities (six probabilities)

Multinomial Logit: Substantive Effects

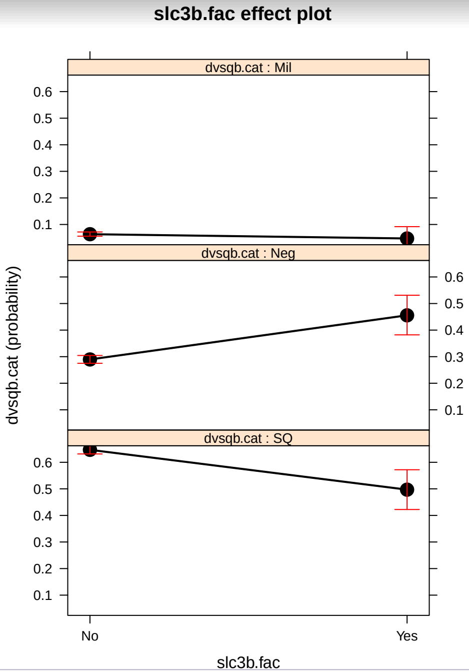
- The IDV we have is a binary variable (can only take 0 OR 1)
- We should only have two sets of probabilities (six probabilities)
- To make `R` realize that this is a binary variable, convert this into a factor variable and re-estimate the model before we use the effect function

Multinomial Logit: Substantive Effects

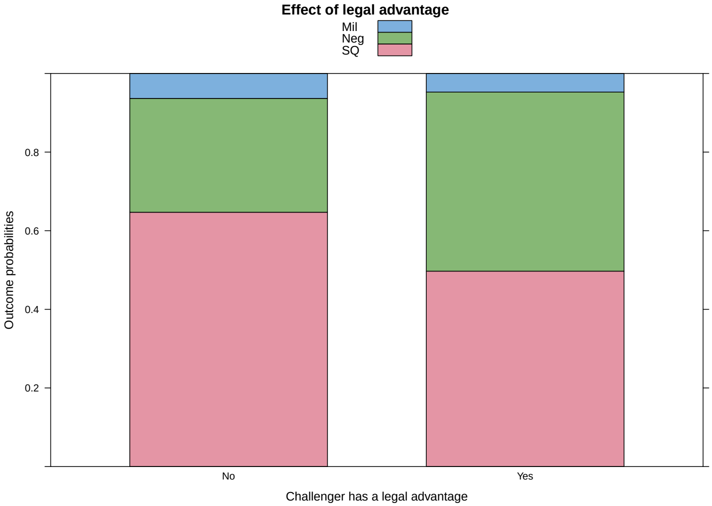
```
slc3b.fac effect (probability) for SQ
slc3b.fac
       No       Yes
0.6469627 0.4970415

slc3b.fac effect (probability) for Neg
slc3b.fac
       No       Yes
0.2895669 0.4556212

slc3b.fac effect (probability) for Mil
slc3b.fac
        No        Yes
0.06347044 0.04733728
```


slc3b.fac effect plot
dvsqb.cat : Mil
dvsqb.cat : Neg
dvsqb.cat : SQ
0.6
0.5
0.4
0.3
0.2
0.1
dvsqb.cat (probability)
No
Yes
slc3b.fac

Multinomial Logit: Substantive Effects



Multinomial Logit: Replication

TABLE 2. Multinomial Logit Analysis of Decision to Challenge the Status Quo

	Negotiations vs. threaten force	*Status quo vs. force*	*Negotiations vs. status quo*
International law			
Strong legal claims	0.744 (0.357)***	0.127 (0.265)	0.617 (0.241)***
Controls			
Democracy	0.794 (0.342)***	0.480 (0.314)†	0.314 (0.115)***
Military balance	−1.310 (0.330)***	−1.114 (0.302)***	−0.196 (0.209)
Common security ties	0.081 (0.203)	0.107 (0.232)	−0.189 (0.121)
Strategic territory	−0.181 (0.252)	−0.210 (0.233)	0.029 (0.136)
Ethnic ties	0.050 (0.207)	−0.243 (0.205)	0.293 (0.115)**
Enduring rivals	−0.836 (0.216)**	−1.130 (0.196)***	0.293 (0.136)**
Constant	2.073 (0.257)†††	2.750 (0.253)	−0.676 (0.142)†††

(*Notes.* $N = 3840$. Clustered standard errors in parentheses.
**$p < .05$, ***$p < .01$ (one-tailed) †$p < .1$, †††$p < .01$ (two-tailed).)

- The authors have other independent variables
- The authors use Mil as the baseline for the first two and SQ as the baseline for the last

Multinomial Logit: Replication

- Strong legal claims: `slc3b.fac`
- Democracy: `demdum`
- Military balance: `milratio`
- Common security ties: `alliance`
- Strategic territory: `strvalue`
- Ethnic ties: `ethvalue1`
- Enduring rivals: `endriv5b`
- Duration control: `sqtime1` **included but not shown**